LOGIC EXERCISES: WEEK 6

QUESTION 1

Suppose you were asked to determine the truth-value of '(P ∧ (Q → P))' in a given L_1-structure *A*. How many sentence-letters would need to be assigned truth-values by that L_1-structure? And what would your answer be if the sentence in question was instead '(P ∧ (Q → R))'?

QUESTION 2

Produce L_2-structures that are counterexamples to the following incorrect claims of validity.

i. ∀x∃y(Px → Qy) ⊨ ∃y∀x(Px → Qy)
ii. (∀xPx ↔ ∀xQx) ⊨ ∀x(Px ↔ Qx)
iii. ∃x((Px ∧ Qx) → Rax) ⊨ ∀x(Rax ∨ (Px ∨ Qx))
iv. ∀x(Px → Rx), ∀x(Qx → Rx), ∃xRx ⊨ ∃x(Px ∧ Qx)
v. ∃x∀y(Rxy ∧ Ryx), ∀x(Px → ∀y(Ryx → Py)) ⊨ Pa
vi. ∀x∀y(Pxy → Pyx), ∀x∀y(Rxy → ∃zPzx), ∃xRxa ⊨ ∀x∀y(Rxy → Pxy)

QUESTION 3

Establish the following claims by producing proofs in natural deduction.

i. (P ∧ Q) ⊢ (Q ∧ P)
ii. ⊢ (P → (Q ∨ P))
iii. (P → Q) ⊢ (¬Q → ¬P)
iv. ⊢ ((P → ¬P) → ¬P)
v. (P ↔ Q), (Q ↔ R) ⊢ (P ↔ R)
vi. (P ↔ Q), ¬Q ⊢ ¬P
vii. ((P ∧ Q) → R) ⊢ (P → (Q → R))
viii. ¬(P → Q) ⊢ P
ix. (P → Q), (¬P → Q) ⊢ Q
x. ((P ∧ Q) ∧ R) ⊢ ((P → S) → ((S → T) → T))

QUESTION 4

Formalize the following argument using L_1, explaining any difficulties you have or any liberties you take in doing so, and use a natural deduction proof to show that it is valid.

> Since John doesn't know much about gardening, he'll only succeed in growing carrots if Mary helps him. After all, to grow them to maturity he will have to get the seeds to germinate and protect the plants from the carrot fly. And while he might succeed in the former by good fortune, without Mary's help he is bound to fail in the latter.